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On scrutiny of the ballot boxes the President declared the following persons duly elected members of the Society.

Prof. Claudio Jannet, of Paris.
Prof. Paul Leroy Beaulieu, of Paris.
M. Emile Malézieux, of Paris.
Prof. E. A. Barber, of West Philadelphia.
Dr. Jas. A. H. Murray, of Mill Hill, London.
Hon. William Butler, Judge of U. S. District Court, Eastern District of Pennsylvania.
Mr. Wm. Woodnutt Griscom, of Philadelphia.

And the meeting was adjourned.

Photodynamic Notes, II. By Pliny Earle Chase, LL.D., Professor of Philosophy in Haverford College.

(Read before the American Philosophical Society, April 15, 1881.)

16. *Weighing the Sun by a Soap Bubble.*

In the well-known experiment of inflating a soap bubble with a mixture of oxygen and hydrogen, and exploding it by a candle, there is an opportunity for studying various radiodynamic relations. The equilibrium, which usually exists between the gravitation of the particles towards the sun and towards the earth, is suddenly and violently disturbed. During the restoration of equilibrium, there are simultaneous tendencies to the production of orbital velocities, about the earth and about the sun.

The height of virtual focal projection which represents elliptic orbital velocity is $\frac{r}{2}(1 + e)$; the height at Earth's equatorial surface, which is in unison with orbital projection, being $\frac{3962.8}{2} \times 1.01677 = 2014.16$ miles. Dividing this height by the French thermal unit $\left(C = \frac{1389.6}{5280}\text{ mile-pounds}\right)$ and multiplying by 9, because 9 pounds of gas are lifted by 1 pound of combustible, we get 68878.2 calorics as the thermal equivalent of the explosion. Naumann* gives the following experimental values :

Thomsen	68376
Favre and Silbermann	68924
Dulong	69486
Hess	69584
Grassi	69332
Andrews	67616
Mean	68886

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* Handbuch der Chemie, p. 290.

The explosive force proceeds radially in all directions from the centre, so that the particles are subjected to cones of force, introducing oscillations which may be represented by a conical pendulum of $\frac{1}{4}$ the height, or 503.54 miles. In seeking equilibrium, the particles tend towards their own centre of gravity, at $\frac{1}{2}$ of $503.54 = 251.77$ m., and also, on account of resistance at the Earth's surface, towards the linear centre of oscillation, at $\frac{2}{3}$ of $503.54 = 335.69$ miles.

The action of Earth on the centre of gravity of the mass produces a secondary centre of oscillation, in which the primary centre of oscillation acts as a point of suspension, and the centre of gravity as a pendulum-extremity of wave propagation. This secondary centre of oscillation is at $\frac{5}{9}$ of $503.54 = 279.745$ m., which represents the mean *vis viva* of oscillatory projection relatively to Earth; the *vis viva* relatively to Sun being represented by Earth's semi-axis major.

Let $r = 3962.8$ miles; $nr =$ Earth's semi-axis major; $t_0 = 1$ year $=$ 31558149 seconds; $t_1 = 2\pi\sqrt{\frac{r}{g}} = 5073.6$ sec.; $m_0 =$ Sun's mass; $m_1 =$ Earth's mass; $h = 279.745$ miles. Then we have

$$\left(\frac{nr}{r}\right)^3 : \left(\frac{t_0}{t_1}\right)^2 :: m_0 : m_1 :: nr : h,$$

$$\therefore m_0 = 331,631\ m_1$$

$$nr = 92,772,200 \text{ miles.}$$

17. *Centres of Density and of Nucleation.*

The rotating photodynamic action which is represented by the equations, $\frac{gt}{2} =$ *velocity of light*, $\frac{gt^2}{4} =$ *modulus*, is shown in its greatest simplicity at the nucleal centres of cosmical systems. In all other places it is complicated with orbital motions, which increase the difficulty of determining the sum of the photodynamic actions which are balanced by an equal sum of gravitating reactions. Doubtless methods will be found hereafter for making the proper allowances in all the most important cases; but at present we can take only a few steps towards the goal. The first step, naturally, is from Sun, the centre of nucleation, to Earth, the centre of density. Here we are helped, as in the first instance, by the study of maxima. The limit between association and dissociation at the nucleal centre is the velocity of light, or the greatest known velocity of wave propagation; the centre of density is revolving about the nucleal centre, and its "nascent" limit shows the greatest possible tendency to circular-orbital velocity in the system.

Earth's nascent velocity, if its orbit were strictly circular, would be $\frac{gt}{2} = \frac{86164}{2} \times \frac{32.088}{5280} = 261.82$ miles. This needs to be multiplied by $(1 + e)^2 = (1.01677)^2$, in order to allow for orbital "subsidence" from aphelion to mean position. We thus get 270.68 miles, for the value of $\sqrt{gr}$ at Sun's equatorial surface. If we knew the precise quotient of Earth's semi-axis major by Sun's semi-diameter, we could easily compute Sun's mass and distance. Different observers give measurements of Sun's diameter, which vary through a range of about a fourth of one per cent.

In expanding or condensing nebulæ, I have shown that Laplace's limiting or atmospheric radius varies as the $\frac{4}{3}$ power of the nucleal radius. Our first step towards transforming Earth's orbital into nucleal efficient velocity, is to multiply $\frac{gt}{2} \times (1 + e)^2$ by $(1 + e)^{\frac{4}{3}}$. We must also multiply by π, because the time of acquiring the efficient velocity is π times as great as the time of acquiring orbital velocity, and by n*, because Sun's equatorial velocity of rotation is n times as great as the corresponding velocity of rotation when expanded to Earth's orbit. We then have :

Earth's $\frac{gt}{2} \times (1 + e)^2 \times (1 + e)^{\frac{4}{3}} \times \pi\, n =$ *velocity of light* $= 10089.15 \times 270.68 \div n^{\frac{1}{2}}$.

The right-hand member of this double equation is obtained as follows :

If $\rho =$ Earth's semi-axis major, its orbital velocity is $2\, \pi\, \rho \div 31558149$; the velocity of light being $\rho \div 497.827$. But 270.68 is $n^{\frac{1}{2}}$ times Earth's orbital velocity ; therefore, the velocity of light is $\left(\frac{31558149}{2\,\pi \times 497.827} = 10089.15\right) \times 270.68 \div n^{\frac{1}{2}}$.

Solving the equation, we find : $n = 214.4775$; $v_\lambda = 186{,}385.7$; $\rho = 497.827\ v_\lambda = 92{,}787{,}850$ miles ; $m_0 = \left(\frac{\rho}{3962.8}\right)^3 \div \left(\frac{1 \text{ yr.}}{5073.6 \text{ sec.}}\right)^2 = 331{,}799\ m_1$.

This purely photodynamic determination of ρ differs by less than $\frac{1}{59}$ of one per cent. from the partially thermodynamic determination in the foregoing note.

18. *Nucleal and Rupturing Centres of Condensation.*

Our next tentative step is to Venus, the nearest and brightest of Earth's sister planets, and the only one which shares with Earth the peculiarity of sometimes having an orbit, in the course of its secular changes of excentricity, which is entirely circular. This fact, together with Earth's central position in the dense belt, indicates a purely nucleal origin for that belt ; the other planets showing more obvious evidences of an origin which was partly atmospheric, and of the influences of "subsidence" which were so happily pointed out by Herschel. The nascent *vis viva* of Venus, $\left(\frac{gt}{2}\right)^2_\beta$, was also a rupturing *vis viva* in the dense belt, as is shown by the following relation between Earth's semi-axis major, ρ_a, and Venus's secular perihelion radius-vector, ρ_β.

$$\left(\frac{gt}{2}\right)^2_a : \left(\frac{gt}{2}\right)^2_\beta :: \rho_a : \rho_\beta \text{ ....} (1.)$$

---

* n represents the quotient of Earth's semi-axis major by Sun's semi-diameter.

Let us take for m_α the value given by Note 17; for m_β, Hill's estimate, $\frac{1}{427840}$; for t_α, $23^h\ 21^m = 84060$ sec. Then

$$\frac{42030}{427840} \times \frac{1}{r_\beta{}^2} = \left(\frac{gt}{2}\right)_\beta \dots\dots (2.)$$

$$\frac{43082}{331799} \times \frac{1}{r_\alpha^2} = \left(\frac{gt}{2}\right)_\alpha \dots\dots (3.)$$

Introducing Stockwell's value, $\rho_\beta = .672241$, together with (2) and (3), into (1), we have

$$\left(\frac{43082}{331799} \times \frac{1}{r_\alpha^2}\right)^2 \times .672241 = \left(\frac{42030}{427840} \times \frac{1}{r_\beta^2}\right)^2$$

Hence we find:

$$r_\beta = .9606\ r_\alpha.$$

Herschel gives .983; Newcomb and Holden, .967; Newcomb, in Johnson's Cyclopædia, .9602; Searle, .96; Chambers, .948.

19. *Centres of Density and Nebulosity.*

Earth's nascent relation to the maximum value of Sun's $\sqrt{gr}$ introduces, as we have seen, the factor of gravitating subsidence to a spheroidal nucleus, $(1 + e)^2$. Jupiter's relation to the same value, indicates the formation of a secondary nucleus, at the centre of primitive nebulosity; for Sun's superficial $\sqrt{gr}$, 270.68, is accelerated, in Jupiter's nascent velocity, in the ratio which is due to solar condensation to Jupiter's secular perihelion, or rupturing atmospheric locus, which is .9397 of its semi-axis major, the greatest eccentricity being .0603. We have, therefore:

$$\left(\frac{gt}{2}\right)_\gamma = (\sqrt{gr})_0 \div .9397 = 288.076 \text{ miles.}$$

Professor Hall's estimate of t_γ is $9^h\ 55^m\ 26^s.5 = 35726.5$ seconds. This gives $g_\gamma = 288.076 \div \frac{t_\gamma}{2} = .016127$ m. $= 2.654\ g_\alpha$; $r_\gamma^2 = \left(\frac{m}{g}\right)_\gamma = \left(\frac{331799}{1047.879} \div 2.654\right) r_\alpha^2$; $r_\gamma = 10.924\ r_\alpha$.

Herschel gives 10.977; Newcomb, 10.85; Searle, 10.93; Chambers, 10.75.

20. *Central Nebulous Rupture and Subsidence.*

The co-ordinate influence which is shown in the nascent velocities of Earth and Venus (Note 18), has an analogue in Jupiter and Saturn. If we represent Jupiter's atmospheric rupturing locus by ρ_γ, and Saturn's locus of incipient subsidence, or secular aphelion, by ρ_δ, we find:

$$\rho_\delta : \rho_\gamma :: (\sqrt{gr})_0 : \left(\frac{gt}{2}\right)_\delta$$

$$10.343 : 4.8863 :: 270.68 : 127.81.$$

Prof. Hall's estimate of t_δ is $10^h\ 14^m\ 23^s.4 = 3683{,}4$ seconds. Dividing 127.81 miles by $\frac{t_\delta}{2}$, we get $g_\delta = .0069344$ m. $= 1.141\ g_\alpha$; $r^2{}_\delta = \left(\frac{331799}{3501.6} \div 1.141\right) r^2{}_\alpha$; $r_\delta = 9{,}113\ r_\alpha$.

Herschel gives 9.87; Newcomb, 8.86; Searle, 9.01; Chambers, 9.07.

21. *Nodal Coördination.*

We see, therefore, from Notes 17–20, that the five nodal positions, which are of the greatest photodynamic importance in the solar system, show successive coördinating influences of universal æthereal oscillation; for, if we designate nascent velocity $\left(\frac{gt}{2}\right)$ by V,

V_0, or the sum of the reactionary gravitating impulses at Sun's surface during a half-rotation, is the velocity of light.

V_α, or the sum of the reactionary gravitating impulses at Earth's surface during a half-rotation, is due to the sum of the reactionary impulses at Sun's surface during the time of communicating orbital velocity, $\sqrt{\frac{r}{g}}$; part of the velocity being compensatory of orbital subsidence, and the rest giving axial rotation.

V_β, or the sum of the reactionary gravitating impulses at Venus's surface during a half-rotation, is due to the belt-rupturing impulses of the reactionary impulses at Earth's surface.

V_γ, or the sum of the reactionary gravitating impulses at Jupiter's surface during a half-rotation, is due, in part, to the sum of the reactionary impulses at Sun's surface during the time of communicating orbital velocity, and in part to the acceleration of Sun's atmospheric condensation to the locus of nebular rupture.

V_δ, or the sum of the reactionary gravitating impulses at Saturn's surface during a half-rotation, is the incipient velocity which, when accelerated by solar atmospheric subsidence to the rupturing locus of its belt, became equivalent to the sum of the reactionary impulses at Sun's surface during the time of communicating orbital velocity.

The joint relations of Earth's eccentricity, Earth's semi-axis major, Sun's semi-diameter, the velocity of light, planetary velocity, nascent velocity of Sun and Earth, and Sun's equatorial velocity of rotation, which are shown in Note 17, are wholly inexplicable upon any hypothesis which fails to recognize the equal actions and reactions of an elastic medium. They are all, however, simple and natural results of photodynamic influence. Sun's nascent velocity is the velocity of light, V_0; the limit of circular-orbital velocity ($V_1 = (\sqrt{gr})_0 = 270.68$ miles), is equivalent to $V_0 \div (1 + e)^{\frac{4}{3}} \pi n$; Earth's nascent velocity $\left(\frac{gt}{2}\right)_a = 261.82$ miles, is equivalent to $V_1 \div (1 + e)^2$; Sun's equatorial velocity of rotation ($V_2 = 1.233$ miles), is equivalent to $\frac{(1 + e)^{\frac{2}{3}}}{n} \times \left(\frac{gt}{2}\right)_a$.

22. *Boundaries of the Dense Belt.*

The uncertainty of Mercury's mass makes any attempt to determine its nascent velocity merely provisional. It seems probable, however, that it bears the same proportion to Earth's nascent velocity as Mercury's radius of incipient subsidence, or secular perihelion (.477), bears to Earth's semi-axis major. If such is the case, it furnishes another instance, like the inner moon of Mars, of the establishment of a nucleolus, by subsidence within a condensing nucleus. If we take Encke's mass, $\frac{1}{4865751}$, and Herschel's time, $t = 24^{\mathrm{h}}\ 5^{\mathrm{m}}$, the proportion

$$.477 : 1 :: \left(\frac{gt}{2}\right)_{\varepsilon} : \left(\frac{gt}{2}\right)_{a}$$

gives

$$r_{\varepsilon} = .379\ r_{a}.$$

Herschel gives .396; Newcomb, .378; Searle, .34; Chambers, .374.

Mars, the outermost of the dense belt of planets, furnishes clear evidence of the dependence of nascent velocity upon photodynamic influence. If we designate the centripetal photodynamic acceleration at Earth's semi-axis major by Φ_{a}, and the action at the mean secular aphelion of Mars by Φ_{η}, we find

$$\left(\frac{gt}{2}\right)_{a} : \left(\frac{gt}{2}\right)_{\eta} :: \Phi_{a} : \Phi_{\eta} :: 1.64416^2 : 1^2$$

Substituting Hall's values for m_{η}, $\left(\frac{1}{3093500}\right)$, and t_{η}, $(24^{\mathrm{h}}\ 37^{\mathrm{m}}\ 22^{\mathrm{s}}.7)$, we get

$$r_{\eta} = .546\ r_{a}$$

Herschel gives .517; Newcomb, .531; Searle, .52; Chambers, .621.

23. *Fourier's Theorem.*

In the activities of the luminiferous æther we may reasonably look for abundant evidences of the truth of Fourier's theorem: “Every periodic vibratory motion can always, and always in one manner, be regarded as the sum of a certain number of pendulum vibrations.”* In Note 5, I gave an estimate of the mass at the centre of condensation, which was based upon this theorem. The same value may also be obtained as follows: If we consider simple lines of force, in homogeneous nebular condensation, we find the centre of gravity coincident with the linear centre, and the centre of oscillation at two-thirds of the distance from a point of suspension, or principal inertia, to a point of oscillation, or inferior inertia. If we consider radial action from or towards a centre, we know that the oscillations of a conical pendulum are synchronous with those of a linear pendulum of four times the length. We know, moreover, that centripetal acceleration varies as the fourth power of the velocity of circular orbital revolution. In the conversion of tendencies to nucleal rotation into tendencies to orbital revolution, the oscillating particles are, therefore, subjected to central influences which may be represented by $(2 \times 3 \times 4)^4 = 331776$, which is the ratio of the mass at the centre of nucleation to the mass at the centre of condensation, as estimated in Note 5. It differs by less than $\frac{1}{22}$ of one per cent. from the value found in Note 16, and by less than $\frac{1}{144}$ of one per cent. from the value found in Note 17.

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* Prof. Mayer's statement of the theorem, in Am. Jour. Sci. [3], viii, 85.

24. *Satellite Velocity at the Centre of Density.*

The influence of Laplace's limit appears in satellite velocity as well as in velocity of rotation (Note 17), at the centre of condensation in the solar system; thus lending added significance to the stellar-solar paraboloids.* If l designates the limit of synchronous rotation and revolution, it is evident, from Kepler's third law, that $\pi\, l^{\frac{3}{2}}\, (gr)^{\frac{1}{2}} = \frac{gt}{2}$, and $\pi^2\, n\, l^{\frac{3}{2}}\, (gr)^{\frac{1}{2}}\, (1 + e)^{\frac{10}{3}} =$ *velocity of light.* If we take Earth's semi-axis major as the unit-length of linear oscillation, the velocity which corresponds to the mean linear *vis viva* is $\left(\frac{2}{3}\right)^{\frac{1}{2}}$; the accelerating limit of satellite velocity divided by velocity of equatorial rotation is $\frac{86164}{5073.6} = 16.9828$; the nascent *vis viva* of nebular atmospheric condensation divided by mean orbital *vis viva* is $[\pi\, (1 + e)^{\frac{4}{3}}]^2$; the *vis viva* of terrestrial projection, in units of nucleal condensation, is n. The projection against uniform resistance, or the synchronous rotating velocity which is due to such projection, is proportioned to the *vis viva*, and we find that

$$\left(\frac{2}{3}\right)^{\frac{1}{2}} : 16.9828 :: \pi^2 \times 1.01677^{\frac{8}{3}} : n = 214.59$$

This value of n differs by less than $\frac{1}{18}$ of one per cent. from the one found by Note 17. Such accordance in the successive transformations of photodynamic energy into energy of rotation and of revolution gives new evidence of the kinetic importance of Earth's position at the centre of belt-condensation.

If we treat e and n as unknown quantities, and find their values by combining the methods of Notes 17 and 20, we get $1 + e = 1.01662$; $n = 214.51$. This value of e differs by less than $\frac{1}{66}$ of one per cent. from the observed value, 1.01677. If we treat n and the constant of aberration as unknown quantities, we find $n = 214.59$; time of light propagation from Sun, $497^s.423$.

25. *Satellite Velocities in the Dense Belt.*

The equation, $\pi l^{\frac{3}{2}}\, (fr)^{\frac{1}{2}} = \frac{ft}{2}$, is true in all cases of radiodynamic centrifugal or centripetal influence, whether cosmical or molecular. The range of uncertainty respecting the masses, densities, and axial rotations of the other planets, is too great to justify any attempts at precise measurement of their dependence upon æthereal or photodynamic influence, but some interesting comparisons may be drawn between the central and collateral energies in each of the great planetary belts, which furnish suggestive indications for future precise observation and analysis. If we refer the satellite velocities of the dense belt to photodynamic energy at the centre of the belt, by taking Earth's limiting velocity as the unit, we may designate the limiting velocities for Mercury, Venus, Earth, and Mars, by v_1, v_2, v_3, v_4, respectively. These velocities vary as $\left(\frac{m}{r}\right)^{\frac{1}{2}}$, and if we take the values of r which were found by comparing the axial periods, with the values of m, which were assumed in comparison (Notes 17, 18, 22), we find :

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* Proc. Am. Phil. Soc., xii, 518; xix, 208; *et al.*

$v_3 : v_1 :: 1 : .4242$	Mercury's semi-axis major	.3871
$v_3 : v_2 :: 1 : (.7255)^{\frac{1}{3}}$	Venus's " "	.7233
$v_3 : v_4 :: 1 : (1 \div 1.502)^2$	Mars's " "	1.5237

All of these ratios point to influences operating within the orbits of the respective planets. The ratio between Earth and Mercury indicates simultaneous nucleolar condensation within a primitive condensing nucleus; the ratio between Earth and Mars indicates the comparative influence of centripetal photodynamic impulsion, or of orbital gravitation towards the Sun; the ratio between Earth and Venus indicates synchronous harmonic influences of projectile *vis viva* within a rotating nebula.

26. *Satellite Velocites in the Rare Belt.*

Extending the foregoing comparison to the lighter planets, we find :

$v_3 : v_5 :: 1 : 5.3838$	Jupiter, mean aphelion	5.4274
$v_3 : v_6 :: 1 : (10.3937)^{\frac{1}{2}}$	Saturn, secular "	10.3433
$v_3 : v_7 :: 1 : 2.0308$	Relative momentum,	29.812
$v_3 : v_8 :: 1 : 1.7271$	" "	29.567

Jupiter, therefore, falls into the same category as Mercury and Earth, as indicating simultaneous nucleolar condensation within a primitive condensing nucleus; the ratio between Saturn and Earth indicates satellite velocities accordant with their respective velocities of nucleal projection; the Uranian and Neptunian ratios multiplied by their respective mass-ratios, point to a primitive unity of momentum near the outer surface of our system in its early stages of condensation.

If we adopt Newcomb's values for the respective masses of Uranus and Neptune, $\frac{1}{22600}$ and $\frac{1}{19380}$, and take Earth's mass, $\frac{1}{331799}$, for the unit of comparison, we find

	Mass.	Limiting sat. v.	m × v.
Earth,	1	1	1
Uranus,	14.68	2.0308	29.812
Neptune,	17.12	1.7271	29.567

Stockwell gives 29.732 for Neptune's mean perihelion, or rupturing locus, so that Earth's relative momentum : the rupturing momentum :: Earth's mean radius vector : the rupturing radius vector. This seems to be a conclusive evidence of a common rotating influence which embraces the centre of nucleation, the centre of density and the Neptuno-Uranian belt.

27. *Harmonic Relations of the Moon.*

In Note 8, the Moon's mass was shown to be in strict accordance with the requirements of radiodynamic elasticity. The two foregoing notes, together with the Moon's position relative to the centre of density, make it probable that its distance from the Earth, as well as its mass, may furnish important harmonic indications. We find, accordingly, the following close approximations :

Sun's distance : Moon's distance : : Jupiter's mean locus of subsidence (mean aphelion) : Sun's radius of condensation : : Neptune's mean rupturing *vis viva* : Earth's nebular *vis viva*. This triple proportionality is satisfied by the following values :

$$92,996,000 : 238,847 :: 1168.07 : 3 :: (19.73)^2 : 1$$

This would give 215.22 for the quotient of Earth's semi-axis major by Sun's semi-diameter, which is probably about $\frac{1}{3}$ of one per cent. too large. These slight differences in the approximations to the value of n may be due to solar atmospheric, photospheric and chromospheric modifications.

The approximation of Moon's semi-axis major, measured in units of Earth's semi-diameter (60.28) to the continued product $3 \times 4 \times 5$, is also noteworthy.

28. *The Moons of Mars.*

The linear-pendulum ratio, 3, which was introduced twice in the foregoing note, appears in a vast number of satellite, planetary, stellar and molecular harmonies. The following table, with its influence upon the moons of Mars, is a very striking instance :

$$3^0 = 1$$
$$3^1 - 3^0 = 2$$
$$3^2 - 3^1 - 3^0 = 5$$
$$3^3 - 3^2 - 3^1 - 3^0 = 14$$
$$3^4 - 3^3 - 3^2 - 3^1 - 3^0 = 41$$

The above numbers are harmonic divisors, in the Mavortian system, of a primitive nebular radius which represents proportionate action between the centre of photodynamic planetary inertia (Saturn) and the locus of internal rupture in the dense belt (Venus's mean perihelion). For we have

Saturn, mean rad. vec. : Venus, mean per. : : 9.539 : .698 : : 13.67 : 1

If we take 13.67 semi-diameters of Mars as the nebular radius, $\frac{1}{2}$ the radius would be the locus of primitive rupturing subsidence, a locus which accords very closely with the semi-axis major of Deimus, the outer satellite. This and other accordances are shown in the following table :

Divisors.	Quotients.	Observed	Positions.
1	13.670	13.670	Nebular radius.
2	6.835	6.846	Deimus.
5	2.734	2.730	Phobus.
14	.976	1.000	Semi-diameter.
41	.333	.333	Linear c. of central oscill'n.

The importance of the ratio of harmonic undulation, which has been thus introduced at the outer limit of the dense belt, has a deeper meaning, when we remember that Venus and Earth are the only two planets whose orbits are ever exactly circular, and therefore the only ones that indicate nebular activities which extend to the very surface of a rotating nucleus.

29. *The Moons of Jupiter.*

We may reasonably look for a greater variety of satellite harmonies in connection with the giant planet which represents the primitive nebular centre of our system, than in connection with any of its subordinate companions. Some of those harmonies have been already indicated in Note 14. It may be well to restate two of them in a different form, together with some others which seem especially significant.

I. If we take axial rotation at the centre of density as the unit of comparison, the synchronous rotation and revolution of the nebular hypothesis has been accelerated 366.2565 fold, by Earth's condensation and other causes. Earth's accelerated rotation, is to the acceleration by fall to a centre of linear oscillation, as Jupiter's synchronous radius is to Callisto's synchronous radius:

$$366.2565 : 9 :: 4332.5848^{\frac{2}{3}} : 16.6891^{\frac{2}{3}}$$

The significance of this rhythmical accordance between the nebular centre and the centre of density is increased by the fact that the nascent velocity at each of those centres is directly traceable to the maximum of planetary velocity ($\sqrt{gr}$ at Sun's surface).

II. The significance is further increased by the fact that Callisto represents the mean centre of gravity of Earth and Jupiter, at Jupiter's rupturing locus. Jupiter's mean perihelion being 4.9782, its distance from Earth's semi-axis major is 3.9782. Jupiter's mass being $\frac{1}{1047.88}$, this relation gives $\frac{1}{331245}$ for Earth's mass, as will be seen in the following proportion, .012585 being Callisto's semi-axis major:

$$3.9782 : .012585 :: \frac{1}{1047.88} : \frac{1}{331245}$$

This estimate of Earth's mass gives 92,736,000 miles for Earth's semi-axis major, and 1,167,100 miles for Callisto's semi-axis major.

III. The harmony of planetary and satellite influences, at the centres of density and of nebulosity, is shown by the proportion,

Jupiter's secular perihelion : Earth's semi-axis major : : Callisto's semi-axis major : Moon's semi-axis major.

4.8863 : 1 : : 1,167,100 miles : 238,850 miles.

Herschel's estimate is		237,000	miles.
Lockyer's	"	238,793	"
Chamber's	"	238,830	"
Von Littrow's	"	238,870	"
Newcomb's	"	240,300	"

IV. An elliptic orbital influence, between Jupiter and the dense belt, with a major-axis of 6.131, is indicated by the proportion:

6.131 : .012585 : : 186282 : 382.38.

The second term of this proportion represents Callisto's semi axis major; the third term, the velocity of light; the fourth term, the velocity acquired by infinite fall to Sun's surface.

V. The pendulum-factor, 3, appears in the Jovian system, in a primitive radius $= 3^3 \times 1.423 = 38.424$ semi-diameters, 1.423 being the theoretical ratio of the radius of constant volume to the radius of constant pressure.*

The following table shows the influence of harmonic vibrations upon the positions of Jupiter's satellites. The harmonic divisor for Callisto is very nearly $1 + 2a$; the other theoretical divisors have a constant third difference of a.

Theoretical Harmonic Divisors.				Observed Divisors.	Quotients.	
		1	$= 1.000$	1.000	48.424	Gaseous radius.
	a					
a		$1 + a$				
	$2a$		1.428	1.423	26.998	Callisto.
$2a$		$1 + 3a$				
	$4a$					
$3a$		$1 + 7a$	2.497	2.503	15.350	Ganymede.
	$7a$					
$4a$		$1 + 14a$	3.994	3.993	9.624	Europa.
	$11a$					
$5a$		$1 + 25a$	6.346	6.353	6.048	Io.
	$16a$					
$6a$		$1 + 41a$				
	$22a$					
$7a$		$1 + 63a$				
	$29a$					
$8a$		$1 + 92a$				
	$37a$					
$9a$		$1 + 129a$				
	$46a$					
		$1 + 175a$	38.424	38.424	1.000	Semi-diameter.

30. *The Moons of Saturn.*

The photodynamic centre of planetary inertia presents the following evidences of the influences of æthereal undulation:

I. Titan, Saturn's giant moon, bears witness to the rupturing tendencies at the centres of nebulosity and of planetary inertia, in the proportion:

Saturn's secular perihelion : Jupiter's mean perihelion : : Titan's semi-axis major : Sun's semi-diameter.

8.7344 : 4.9782 : : .00817 : .00466.

II. The combined influences of orbital revolution at the centre of density, and of Laplace's tendencies to synchronism, are shown in the approximate equivalence of the terrestrial year to 23 orbital revolutions of Titan.

$366.2565 = 23 \times 15.924$; Titan's orbital period $= 15.945$ days.

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* Proc. Am. Phil. Soc. xiv, 651.

III. Saturn's satellite system presents two sets of harmonic indications, Titan's mass being so great as to establish an independent series of divisors.

	Distance.	Divisors.	Harmonic Divisors.	
Japetus,	64.359	1.000	1.000	
Hyperion,	26.795	2.402	2.397	$= 1 + a$
Rhea, aphelion,	9.747	6.603	6.588	$1 + 4a$
Dione,	6.821	9.436	9.384	$1 + 6a$
Tethys,	5.325	12.087	12.179	$1 + 8a$
Enceladus,	4.300	14.966	14.974	$1 + 10a$
Mimas,	3.351	19.206	19.166	$1 + 13a$
Japetus,	64.359	1.000	1.000	
Titan,	22.084	2.914	2.920	$= 1 + b$
Rhea,	9.526	6.756	6.760	$1 + 3b$
Semi-diameter,	1.000	64.359	64.359	$1 + 33b$

31. *The Moons of Uranus.*

The outer moon of Jupiter, the giant moon of Saturn, and the inner moon of Uranus are the ones which give the most direct testimony to the harmonic influence of planetary oscillations. The primary satellite-harmony is determined by the joint influence of Jupiter, Earth and Sun, in the systems of Earth and Jupiter; Saturn, Venus and Sun, in the system of Mars; Jupiter, Saturn and Sun, in the Saturnian system; Jupiter, Uranus and Sun, in the system of Uranus. The nucleal centre has, therefore, modified all the systems; the centre of nebulosity, all but the systems of Mars and Neptune; the centre of planetary inertia, its own system and that of Mars; the centre of condensation, its own system and that of Jupiter. The influence of each planetary radius appears in its own system; the influence of the radius vector also appears, except in the case of Mars, the outer planet of the dense belt, which shows the interaction of mean planetary inertia in the outer belt and internal rupturing tendencies in the inner belt.

I. The mean rupturing locus of Uranus : Jupiter's mean locus of subsidence : : Sun's semi-diameter : Ariel's semi-axis major.

18.323 : 5.427 : : .00466 : .00138.

II. The outer moon of the Uranian system represents, within less than two per cent., the retarded extremity of a linear pendulum, of which the inner moon represents the centre of oscillation. The harmonic divisor for the second moon, as in the Jovian system, is of the form $1 + 2a$. The subsequent differences are multiples of 5, the range between $1 + 2a$ and $1 + 127a$ being $5^3 a$. The harmonic details are shown in the following table:

	Distance.	Divisors.	Harmonic Divisors.	
Oberon,	22.75	1.000	1.000	
Titania,	17.01	1.337	1.343	$= 1 + 2a$
Umbriel,	10.37	2.194	2.199	$1 + 7a$
Ariel,	7.44	3.058	3.055	$1 + 12a$
Semi-diameter,	1.00	22.750	22.750	$1 + 127a$

32. *Neptune's Moon.*

Is Neptune the outer limit of our planetary system, or may we look for still more distant members of our family of circling orbs? Astronomers find indications of some further source of remote perturbation, and attempts have already been made by Prof. George Forbes and D. P. Todd to furnish a clue to its discovery. More than eight years ago, I communicated to the American Philosophical Society* a planetary series which introduced the ratio of the circumference of a circle to its diameter, which is also the photodynamic ratio, at Sun's surface, between the reactions which represent the velocity of light and the reactions which represent the maximum of planetary velocity at Laplace's limit. The two terms of the series which immediately follow Neptune are 94.38 and 296.52 times Earth's semi-axis major. Forbes, by investigating the perturbations which are indicated by the aphelia of comets, has strikingly corroborated this forecast. The abstract of his memoir read before the Royal Society of Edinburgh says†: "On tabulating the aphelion distances of all the known elliptic orbits of comets, it was found that in no case was there any grouping of aphelion comet distances which did not agree with the distances of planets, except that beyond the distance of Neptune there were two groupings of comet aphelion-distances, one at 100 Earth's radii, the other at 300 Earth's radii, approximately." The perihelion or rupturing positions which are indicated by his investigations are 96.7 and 285.2.

Both of these investigations were entirely independent of my own, and neither of the writers gives any indication of ever having read my paper. There is, therefore, cumulative and conclusive evidence of harmonic influences, beyond the known limits of the planetary system as well as between the interior limits of the system and the Sun. The similarity of those influences, their dependence upon photodynamic action, and their importance as guides to investigation, are shown in the following note (33).

Neptune's satellite-harmony is the only one which involves the consideration of only two of the members of the system, Sun and itself. This fact, as well as the slight secular eccentricity of Neptune's orbit, seems to render it probable that the supra-Neptunian harmonies may indicate asteroidal rather than planetary aggregation. If Neptune represents the primitive belt of subsidence-collision, the corresponding nebular radius was $\frac{3}{2} \times$ its secular aphelion radius-vector, or 45.704 × Earth's semi-axis major. We find, accordingly, that

I. Sun's mass : Neptune's mass : : Nebular radius : Neptune's lunar radius-vector.

$$19380 : 1 : : 45.704 : .00236.$$

II. Sun's semi-diameter : Neptune's lunar radius : : Modulus of light : virtual projection of light-velocity, *very nearly; i. e.*, within 1.3 per cent.

$$.00466 : .00236 : : 1 : .5065.$$

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* Proceedings A. P. S., xiii, 140; Feb. 7, 1873.

† The Observatory, No. 38, p. 441.

33. *Infra-Mercurial Photodynamic Nodes.*

In my original anticipation* of harmonic nodes between Mercury and the Sun, I looked for possible unknown planets, planetoid groups, or other seats "of solar and planetary perturbation." The announcement † to the Royal Society, seven weeks afterward, of a sun-spot period which was synchronous with the revolution of the first infra-Mercurial node, furnished conclusive evidence of the predicted perturbations; the observation of supposed planetoid bodies, by Watson and Swift, rendered it probable that some of the perturbation is due to actual planetoid influences; the discussions of planetoid sun-spots, by Kirkwood, Mouchez, Gaillot, and Von Oppolzer, indicated four additional periods ‡ in close accordance with four other harmonic nodes; a sixth node (.139) marks the locus (.140) of orbital time, which is a mean proportion between Earth's day and year; the six inner nodes have important photodynamic relations which merit special consideration.

The *vis viva*, and consequently the modulus of the velocity of any revolving particle, varies inversely as the radius vector. Hence, the photodynamic modulus of aggregating velocity (M_λ) : modulus of reacting mass (M_μ) : : radius of reacting mass (r_μ) : radius of aggregating velocity (r_λ). The quotient of modulus by the radius of rotation is equivalent to $(\pi\ \nu)^2$, ν being the quotient of $\sqrt{gr}$ by the velocity of rotation. The controlling bodies of the system are Sun and Jupiter, and if we take Jupiter's mass as the revolving unit of mass, Sun, the reacting mass, will be represented by 1047.879. If n designates the quotient of Earth's semi-axis major by Sun's radius (r_μ), $M_\lambda = \left(\frac{31558149}{497.827 \times 2\ \pi\ n^{\frac{1}{2}}}\right)^2$; $M_\mu = (^\pi \times 1047.879)^2$. If $n = 214.5$, $M_\lambda = (688.873)^2$; $\nu = 688.873 \div \pi = 219.28$: $r_\lambda = M_\mu \div M_\lambda = 22.84$; the time of revolution at Sun's surface $(t_0) = 31558149$ sec. $\div\ 214.5^{\frac{3}{2}} = 10045.5$ sec.; the time of solar half-rotation, or the time in which gravitating reaction $\left(\frac{gt}{2}\right)$ against photodynamic action equals the velocity of light, $(t_\lambda) = \frac{\nu}{2} t_0 = 12.747$ days; r_λ is the locus which I have designated as Helios,§ or the distance at which a planet would revolve in t_λ. This gives, for the fundamental radius of my harmonic forecasts, 3.5134 × Earth's semi-axis major, which exceeds Jupiter's mean locus of subsidence collision ($\frac{2}{3}$ of 5.2028) by less than 1.3 per cent. The closeness of the independent verifications, by various observers, is shown in the following table:

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* Proc. Am. Phil. Soc., xiii, 238; New York Tribune, May 2, 1873.

† Proc. Am. Phil. Soc., xiii, 470.

‡ Ib. xviii, 35, 209.

§ Proc. Am. Phil. Soc., xviii, 35.

Harmonic Prediction.		Calculation from Observation.	
$\frac{1}{1}$	= 3.513	Node of subsidence,	3.513
$\frac{1}{5}$	.703	Venus, mean perihelion,	.698
$\frac{1}{9}$	.390	Mercury,	.387
$\frac{1}{13}$	.270	De LaRue, S. and L.,	.267
$\frac{1}{17}$	.207	Kirkwood,	.209
$\frac{1}{19}$	.185	Gaillot, (Watson, II)	.180
$\frac{1}{21}$	.167	Gaillot & M., (Watson, I)	.164
$\frac{1}{25}$	.141	Chase,	.140
$\frac{1}{29}$	.121	Von Oppolzer,	.123

I. The first of the six inner nodes, is $r_\lambda = 22.84$ Sun's semi-diameters, or nearly $= \frac{1}{33}$ Node of subsidence $= .1065$ Earth's semi-axis major. The others are found as follows:

II. $1 \div 366.2565^{\frac{2}{3}}$ $= .0195$ Earth's semi-axis major.
III. $(9^h\ 55^m\ 26^s.5 \div 1 \text{ year})^{\frac{2}{3}}$ $= .0109$ " " "
IV. $2^{\frac{2}{3}} \div 214.5$ $= .0074$ " " "
V. $\left(\frac{4}{3}\right)^{\frac{2}{3}} \div 214.5$ $= .0057$ " " "
VI. $1 \div 214.5$ $= .0047$ " " "

All of these nodes represent synchronous oscillations which are due to mutual actions and reactions among the three important centres of nucleation (Sun), condensation (Earth), and nebulosity (Jupiter). While they are all, unquestionably, radiodynamic, their simple dependence upon the equality between Sun's nascent velocity and the velocity of light obviously refers them to the photodynamic branch of radiodynamics. Can any good reasons be given for regarding them as in any sense thermodynamic or electrodynamic?

The accordance of these six inner nodes of synchronism with the harmonic nodes, is shown in the following table:

	Harmonic nodes.		Synchronous nodes.	
I.	$\frac{1}{33}$	= .1065	.1069	Velocity of light.
II.	$\frac{1}{177}$	.0199	.0195	Earth's day.
III.	$\frac{1}{321}$	.0109	.0109	Jupiter's day.
IV.	$\frac{1}{465}$	.0076	.0074	Two revolutions at r_μ.
V.	$\frac{1}{609}$	.0058	.0057	Solar subsidence.
VI.	$\frac{1}{753}$	.0047	.0047	Sun's semi-diameter.

34. *Photodynamic Projection of Hydrogen.*

The principles which are involved in Note 16 and in the first synchronous node of the foregoing note, are further illustrated by the simple relations which the molecular velocity of hydrogen bears to the velocity of light (v_λ), the velocity of equatorial solar rotation (v_r), and Earth's secular eccentricity. The reaction of gravitation against photodynamic action, at the centre of any stellar system, requires that

Modulus of light : Stellar semi-circumference :: $v_\lambda : v_r$.

The projections of hydrogen in solar explosions, together with the many indications that hydrogen represents the first known stage of æthereal condensation, make it probable that Earth's orbital projection $\left(p = \frac{1+e}{1-e}\right)$, and the molecular velocity of hydrogen (v_h) may also be directly dependent upon photodynamic action. Such dependence is shown by the equation

$$p\, v_h = v_r.$$

The secular value of p, according to Stockwell, is $\frac{1.0338}{.9662} = 1.06994$; molecular velocity of hydrogen v_h, at the temperature of maximum water-density, is $\sqrt{\frac{277}{273}} \times 6050 = 6094.14$ ft. $= 1.1542$ miles; hence v_r, or Sun's equatorial velocity of rotation, $1.06994 \times 1.1542 = 1.235$ miles. Multiplying by the number of seconds in 12.747 days (t_λ, see foregoing note) and dividing by π, we get 432,931 miles for Sun's semi-diameter, and $214.5 \times 432{,}931 = 92{,}863{,}700$ miles for Earth's semi-axis major. These relations, also, are photodynamically radiodynamic, but it would be difficult to find in them any evidence of thermodynamic or electrodynamic action.

35. *Æthereal, Gaseous and Planetary Densities.*

I have already given two approximations* to the comparative æthereal ratio of elasticity to density, and one approximation† to the relative densities of æther and hydrogen. The accumulating evidences of universal photodynamic energy, leading to various accordant æthereal, gaseous and cosmical harmonies, are such as to justify another tentative step in the same direction. From chemical analogy the planetary aggregations, at the centres of nebulosity (μ_5) and of density (μ_3), may well be presumed to give indications of the comparative energies of condensed and of uncondensed æther. We may, therefore, hypothetically assume that

$\mu_5 : \mu_3 ::$ Projectile gaseous *vis viva* : Æthereal *vis viva.*

The quotient of Sun's mass (μ_0) by Earth's mass (μ_3), which is indicated by the foregoing note, is 332,613. This gives, for the quotient of Jupiter's mass (μ_5) by μ_3, 317.416, and for the velocity of light, 186,538 miles. Substituting these values in the above proportion, and designating the densities of æther and of hydrogen by δ_a and δ_h, we get

$$\frac{317.416}{1} = \frac{\delta_h}{\delta_a} \times \left(\frac{1.235}{186538}\right)^2$$

$$\delta_h = 7{,}242{,}500{,}000{,}000\ \delta_a$$

This exceeds the ratio which was deduced from æthereal and gravitating interaction† by a little more than six per cent. But in that approximation the factor of terrestrial projection, $p = 1.06994$, was omitted. If we introduce this factor and increase our estimate of Earth's density about $\frac{7}{9}$ of one per cent., the accordance between the two independent approximations will be exact. If the æthereal density towards the centre of density varies inversely as the square of the distance from Sun, the ratio at Earth's orbital distance should be 333,230,000,000,000,000, representing about two and a half times the æthereal density which Sir William Thomson gives as his lowest estimate.

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* Proc. Am. Phil. Soc , ix, 440; xii, 407.

† Ib., xix, 209.

36. *The Experimentum Crucis.*

At the meeting of the American Philosophical Society in which Dr. Henry Draper announced his discovery of oxygen in the Sun, he proposed that I should test my theory of harmonic æthereal action by an examination of the solar spectrum. I accordingly sent a note to the Society, on the 23d of August, 1877,* in which I showed that the wave-lengths of five of the Fraunhofer lines were in harmonic progression, and that four of them were also figurately harmonic, viz :

Lines.	Wave-lengths.	Divisors.	Harmonic Quotients.
A	761.20	1.0000	761.20
B	687.49	1.1071 = 1 + 7 × .0153	687.56
D_1	589.74	1.2907 = 1 + 19 × .0153	589.76
F	486.52	1.5661 = 1 + 37 × .0153	486.05
H_1	393.59	1.9333 = 1 + 61 × .0153	393.73

The cosmical importance of these harmonies is shown by their relation to the projectile energies of Sun (μ_0) and Jupiter (μ_5), as measured by the product of their respective masses by their perihelion vector-radii. For

$$\frac{\mu_5\,\rho_5 - \mu_0\,\rho_0}{\mu_0\,\rho_0} = .0153.$$

I showed, moreover, that the combined photodynamic influence of the centres of nucleation and of nebulosity, μ_0 and μ_5, is traceable in the remaining Fraunhofer and other subordinate lines, viz :

Lines.	Wave-lengths.	Divisors.	Harmonic Quotients.
C	656.67	1.1530 = 1 + 10 × .0153	660.19
	634.05	1.1989 = 1 + 13 × .0153	634.92
	550.70	1.3825 = 1 + 25 × .0153	550.60
E	527.38	1.4437 = 1 + 29 × .0153	527.26
b	517.70	1.4743 = 1 + 31 × .0153	516.31
	458.66	1.6579 = 1 + 43 × .0153	459.13
G	431.03	1.7650 = 1 + 50 × .0153	431.27

The influence of the pendulum-factor is shown in the tendency to differences among the harmonic divisors, which are multiples of 3 × .0153 or sub-multiples of $2^2 \times 3 \times .0153$.

37. *Supra-Telluric Æthereal and Cosmical Nodes.*

The crucial demonstration of the law of harmonic oscillation, in the æthereal medium as well as in the bodies which are whirling in its eddies, gave conclusive evidence of the importance, in all physical investigations, of studying elastic reaction as well as centripetal or centrifugal action. An important fact, in connection with such comparative study, is the variation of elastic density in inverse geometrical ratio, when distance varies in arithmetical ratio. We may, therefore, look for exponential harmonic relations between planetary positions, of a character analogous to the simple harmonic relations between wave lengths. Let k_a represent the kinetic projectile ratio of Neptune, Sun's semi-diameter being the unit; k_β, the mean projectile ratio of the nebular centre of planetary inertia; k_γ, the projectile ratio of the centre of spherical gyration of the centre of condensation; λ_a, λ_β, etc., the corresponding wave-lengths; $t\chi$, the ratio of orbital time; t_λ, the ratio of photodynamic time; v_λ, orbital velocity at the rupturing locus of mean planetary inertia; $v\chi$, orbital velocity at the subsidence locus of mean planetary inertia; n, the quotient of Earth's semi-axis major by Sun's semi-diameter. Then we have:

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* Proc. xvii, 109–12.

$$\log.\ k_a : \log.\ k_\beta :: \lambda_a : \lambda_\beta \ldots\ldots (1)$$
$$\log.\ k_a : \log.\ k_\gamma :: \lambda_a : \lambda_\gamma \ldots\ldots (2)$$
$$\lambda_\beta : \lambda_\gamma :: 3^2 : 4^4 \ldots\ldots (3)$$
$$t\chi : t_\lambda :: v_\lambda : v\chi \ldots\ldots (4)$$

These four proportions furnish data for estimating the four photodynamic values which are least known, by substituting the six which are best known. In equation (1), the estimates of k_a range between 30.034 n (Stockwell) and 30.070 n (Newcomb), the estimates of n varying from 214.3 to 214.9. As the uncertainty of n is the greater, I adopt it as one of the four unknown quantities, and use Leverrier's intermediate coefficient, 30.067. For λ_a I take the wave-length of the Fraunhofer line A, 7612 ten millionths of a millimeter. In equation (2), in the æthereal sphere of which the centre of condensation is the nucleus, the mean projection of spherical gyration is represented by .4 n, and the radius of solar reaction (k_γ) by .6 n. In equation (3), the numbers 3 and 4 are linear representatives of the exponential radii of synchronous nucleal and atmospheric nebular variation; 3^2 is the pendulum ratio of varying central force; 4^4 is the ratio of variation in the centripetal energy of nodal collisions in paraboloidal aggregation. In equation (4), $t\chi = (30.037\ n)^{\frac{3}{2}}$; $t_\lambda =$ the solar modulus of light divided by Sun's semi-diameter $= \left(\frac{31558149}{2\ \pi\ n^{\frac{1}{2}} \times 497.827}\right)^2 = 101790260 \div n$; $v_\lambda \div v\chi = \left(\frac{1+e}{1-e}\right)^{\frac{1}{2}}$, e being the maximum secular eccentricity of the planetary aggregation (Saturn) at the nebular centre of planetary inertia. Making these substitutions, we get,

$$\log.\ 30.037\ n : \log.\ k_\beta :: 7612 : \lambda_\beta \ldots\ldots (1)_1$$
$$\log.\ 30.037\ n : \log.\ .6\ n :: 7612 : \lambda_\gamma \ldots\ldots (2)_1$$
$$(30.037\ n)^{\frac{3}{2}} : 101790260 \div n :: (1 + e)^{\frac{1}{2}} : (1 - e)^{\frac{1}{2}} \ldots\ldots (4)_1$$

Solving the proportions, we find the following accordances :

Theoretical.		Observed.		Ratio of Difference.
n	214.45	214.45	Brit. Naut. Almanac.	0 of one per cent.
k_β	1.1862	1.1842	Stockwell,	$\frac{1}{6}$ “ “
λ_β	148.20	148.18	Lockyer,	$\frac{1}{74}$ “ “
λ_γ	4215.55	4215.00	Lockyer,	$\frac{1}{74}$ “ “

$$(1+e)^{\frac{1}{2}} : (1-e)^{\frac{1}{2}} : : v_\lambda : v_\chi : : (1.0852)^{\frac{1}{2}} : (.9148)^{\frac{1}{2}} : : (1.1862)^{\frac{1}{2}} : 1 : : (k_\beta)^{\frac{1}{2}} : 1$$

$$k_a = 6441.4$$
$$k_\gamma = 128.67$$
$$\lambda_0 = 16.467$$

The same principles are further illustrated in the following comparative table :

Exponents and Divisors.		Harmonic Roots.	Observed.		Harmonic Quotients.	Observed.
1.0000		6441.4	6441.4	Neptune,	7612	7612 A.
1.05355 =	1 + $\frac{7}{2}$ × .0153	4125.1	4113.9	Uranus,	7225	7196 a.
1.1071	1 + 7 × .0153	2757.7	2742.6	“ c. o.,	6876	6875 B.
1.1530	1 + 10 × .0153	2011.8	2045.6	Saturn,	6602	6567 C.
1.1989	1 + 13 × .0153	1503.5	1510.7	Geom. mean,	6349	6340
1.2448	1 + 16 × .0153	1148.0	1115.7	Jupiter,	6257	6236
1.2907	1 + 19 × .0153	893.6	892.6	$\frac{4}{5}$ “	5898	5897 D.
1.3366	1 + 22 × .0153	707.6	707.6	(167) (168)	5695	5687
1.3825	1 + 25 × .0153	569.1	569.2	(85)	5506	5507
1.4284	1 + 28 × .0153	464.1	457.2	(149)	5329	5274 E.
1.4743	1 + 31 × .0153	383.4	383.3	Mars,	5163	5177 b.
1.5202	1 + 34 × .0153	320.3	326.8		5007	5011
1.5661	1 + 37 × .0153	270.5	270.3		4860	4865 F.

The “exponents and divisors” are terms of an arithmetical progression, as was shown in the foregoing note, the common difference (.0153 ; Note 36) representing a differential action of Sun and Jupiter, the two controlling masses of the system. Therefore, all the roots which are obtained from 6441.4 by using the series as exponential denominators, are harmonic roots, and all the quotients which are obtained from 7612, by using the series as divisors, are harmonic quotients. The “observed” roots represent the mean planetary distances, if $n = 214.45$; the “observed” wave-lengths are given by Gibbs, in the American Journal of Science,* except *a*, which is interpolated from Angstrom's table. It is, therefore, evident that the external planetary positions and the most prominent spectral lines are both influenced by the same harmonic oscillations in the æther, modified by the law of density in the planetary nodes, and by the law of simple distance in the excursions of wave-lengths.

Harmonic influence is also shown in the roots which correspond to Uranus's linear centre of oscillation, the geometrical mean between the

* [2] xliii, 4.

centres of nebulosity and of nebular planetary inertia, $\frac{4}{5}$ of Jupiter, the mean distance of asteroids 167 and 168, asteroid 85 and asteroid 149. The four roots between 893.6 and 464.1 are all asteroidal, the centre of asteroidal density being represented by 569.1. Newcomb gives the mean distances of 172 asteroids, of which 104 are at mean distances ranging between 506 and 594. The curve of frequency has a maximum at 566; the mean distance of the four chief asteroids in point of magnitude, Ceres, Juno, Pallas and Vesta, is 567; the mean distance of the five which include the maximum and are limited by equal ordinates of frequency, is 569.1. The mean distance of Mars is an arithmetical mean between 383.3 and 270.3, which correspond very nearly with the secular apsides.

38. *Nodes in the Dense Belt.*

In my harmonic anticipation of intra-Mercurial planets,* I showed that the controlling influence of Jupiter is modified by the nodal influences at the centre of the dense belt. This modification is also shown, and perhaps more strikingly, in the relations between the æthereal and cosmical nodes within the belt. If we begin with Mars, at the outer limit of the dense belt, and take successive increments of $\frac{15}{2} \times .0153$, we find the following accordances:

Exponents and Divisors.		Harmonic Roots.	Observed.		Harmonic Quotients.	Observed.
1.5202 =	$1 + \frac{68}{2} \times .0153$	320.3	326.8	Mars,	5007	5011
1.6350	$1 + \frac{83}{2} \times .0153$	213.6	214.4	Earth,	4656	4671
1.7497	$1 + \frac{98}{2} \times .0153$	150.3	149.7	Venus, m. p.,	4350	4310 G.
1.8644	$1 + \frac{113}{2} \times .0153$	110.4	110.7	" l. c. o.,	4083	3972 H.
1.9792	$1 + \frac{128}{2} \times .0153$	84.0	83.0	Mercury,	3846	
2.0939	$1 + \frac{143}{2} \times .0153$	65.9	65.9	" p.,	3635	
2.2087	$1 + \frac{158}{2} \times .0153$	53.0	55.3	" l. c. o.,	3446	

The infra-Telluric loci which represent the harmonic roots are Venus's mean perihelion; Venus's linear centre of oscillation of incipient subsidence, or of secular aphelion; Mercury, semi-axis major; Mercury, perihelion; Mercury, linear centre of oscillation. All of the harmonic roots, both supra-asteroidal and infra-asteroidal, correspond *precisely* with planetary loci, and *very nearly* with apsidal or mean positions. All the harmonic quotients represent regions of spectral interference, and correspond very nearly with the mean positions of important spectral lines. The numerous evidences which have been adduced of the influence of planetary disturbances upon Sun spots, together with Lockyer's explanation of the spots by "the approach or retreat of vapor," seem to justify a careful and systematic study of the variations in the position and breadth of spectral lines, such as I recommended in my note of August 23, 1877.† By observations at different hours of the day, it would soon be found whether there is any systematic daily variation; three months would suffice for approximately estimating the disturbing influence of Mercury; and in a year all the planetary regions of the spectrum might be studied. The relations of the C line to Saturn and to hydrogen may, perhaps, be found specially interesting.

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*Proc. Am. Phil. Soc., xiii, 237-9.

†Ib. xvii, 112.

39. "*Basic Lines.*"

The values of λ_0 and λ_γ (Note 37) serve to connect Lockyer's basic-lines with æthereal and cosmical nodes.

Harmonics.	Basic Lines.
$4^4 \times \lambda_0 = 4215.6$	4215
$(4^4 + 7^2) \times \lambda_0 = 5022.4$	5017
$(4^4 + 7^2 + 3^2) \times \lambda_0 = 5170.6$	5170
$(4^4 + 7^2 + 3^2 + 2^2) \times \lambda_0 = 5236.5$	5235
$(4^4 + 8^2) \times \lambda_0 = 5269.4$	5269
$(4^4 + 8^2 + 3^2) \times \lambda_0 = 5417.6$	5416
$4^2 \times 5^2 \times \lambda_0 = 6586.8$	6567

The third and fifth "basic lines" are double. Lockyer gives for the third, b_3, b_4; for the fifth, 5268, 5269. Gibbs gives 5177 for the length of the b line; Angstrom's table gives 5167, 5164, for b_3, b_4. The mean, according to Gibbs' measurements, is 5170. The three hydrogen lines, α, β, γ, correspond nearly with the Fraunhofer lines, C, F, and G, but Angstrom makes the wave-length of the γ line 4340. The three lines may be expressed by $4^2 \times 5^2 \times \lambda_0 = 6586.8$; $(4^4 + 5 \times 2^3) \times \lambda_0 = 4873.7$; $(4^4 + 2^3) \times \lambda_0 = 4346.7$.

40. *Chemical Spectra.*

Soon after my announcement of the accordance between the simple harmonic wave-lengths in the solar spectrum and the exponential harmonic loci in the solar system, I showed * that the law of harmonic vibration also extends to the spectra of chemical elements. The following accordances are especially interesting in relation to the basic lines:

	Normal Differences.	Observed.	
Hg.,	$5428.0 + 2\,\lambda_0 = 5460.9$	5428.0	5461.3
Pb.,	$4390.7 + 6\,\lambda_0 = 5378.7$	4390.7	5377.1
Li.,	$4599.3 + 12\,\lambda_0 = 4796.9$	4599.3	4794.8
Rh., Ir.	$5305.2 + 9\,\lambda_0 = 5453.6$	5305.2	5454.4
Cu.,	$5292.7 + 30\,\lambda_0 = 5786.7$	5293.0	5786.7
As.,	$5335.5 + 51\,\lambda_0 = 6174.7$	5335.5	6175.4
	$5787.3 + 20\,\lambda_0 = 6116.6$	5787.3	6116.7
Zn.,	$4723.2 + 84\,\lambda_0 = 6106.4$	4722.5	6106.4
	$6106.4 + 16\,\lambda_0 = 6369.9$	6106.4	6369.9

The tendency to composite and pendulum multiples of the fundamental line is also shown here; 2, 2×3, $2^2 \times 3$, 3^2, $2 \times 3 \times 5$, $3 \times (2^4 + 1)$, $2^2 \times 5$, $2^2 \times 3 \times 7$, 2^4. The Arsenic line, 5335.5, is $2^2 \times 3^4 \times \lambda_0$; in the two copper lines which precede the two which I have given, the differences are $\frac{1}{7}$ of 30 λ_0 and $\frac{8}{7}$ of 30 λ_0.

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* Ib. xvii, 297–301.

41. *The Corona Line.*

In the communication* which led to my spectroscopic investigations, Dr. Draper anticipated the probability of finding a key to "the 1474 K or corona line." The following accordances may help towards a solution of the mystery. The numbers in the right-hand column are the measurements by Gibbs:

Geometric Wave-lengths.	Observed	Wave-lengths.
$4311.9 \times 1.0305^{14} = 6567.8$	C	6566.7
$4311.9 \times 1.0305^{7} = 5321.7$	1474 K	5322.0
$4311.9 \times 1.0305^{4} = 4862.8$	F	4865.2
$4311.9 \times 1.0305^{0} = 4311.9$	G	4310.3

I have shown† that the harmonic densities of C and G are photodynamically represented by the loci of Saturn and Venus. The corona line is a mean proportional between C and G. The exponential ratio of $\frac{F}{G}$ to $\frac{C}{F}$ is the same as the ratio of the mean moment of inertia of spherical rotation to the moment of superficial equatorial rotation. The ratio of $\frac{1474\ K}{F}$ to $\frac{F}{G}$ is the same as the ratio of variation in an expanding or contracting nucleal radius to the atmospheric radius.

42. *The Hydrogen Lines.*

Lockyer gives four hydrogen lines, C, F, γ, h. The corresponding geometric wave-lengths give the equations, $Fh^2 = \gamma^3$; $F^{11} = C^4h^7$.

Geometric Wave-lengths.	Angstrom's	Wave-lengths.
$4099.5 \times 1.01435^{33} = 6560.2$	C	6561.8
$4099.5 \times 1.01435^{12} = 4863.9$	F	4860.6
$4099.5 \times 1.01435^{4} = 4339.9$	γ	4340.
$4099.5 \times 1.01435^{0} = 4099.5$	h	4101.2

The most prominent line in the hydrogen spectrum, and the only one in some cases, is F, which, as I have shown, is geometrically connected with the corona line and with important cosmical nodes.

43. *Three Orders of Spectra.*

The two foregoing notes introduce a new order of spectra which seems to be of a more elementary character than either of the two which I have previously considered. The geometrical progression of wave-lengths suggests a repetition of impulse, while the absence of some of the terms of the progression indicates a want of complete homogeneity.

The second order is that of the basic lines, which consists of a combination of arithmetical repetitions of a fundamental wave-length, which is harmonically related to cosmical nodes.

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* Proc. Am. Phil. Soc., xvii, 77.

† Ib., xvii, 109–12.

The third order is the one which first attracted my attention,* consisting of groups of wave-lengths which are in harmonic instead of geometric progression, and indicating, like the first order, a want of complete homogeneity.

These three orders are variously blended in the spectra of so-called chemical elements. The character of the blending may, perhaps, serve as a guide towards the resolution of the spectra into simpler constituents.

44. *Æther, Corona, Hydrogen.*

In a communication to the Society, on the 18th of December, 1863, I showed that there is some elastic influence between Sun and Earth, which enables us to form approximate estimates of Sun's mass and distance. At the close of the communication was the following paragraph.†

"The revolution of the Sun around the great Central Sun must also cause barometric fluctuations that may possibly be measured by delicate instruments and long and patient observations. The Torricellian column may thus become a valuable auxiliary in verifying or rectifying our estimates of the distances and masses of the principal heavenly bodies."

In various subsequent communications I have shown that electricity, magnetism, solar gravitation and rotation, planetary and stellar positions, spectral lines, chemical affinity, thermal energy, and other physical manifestations,‡ furnish marked indications of an all-pervading elastic medium, vibrating with the velocity of light, and subject to the same laws of harmonic nodal action as have been found to influence the air and other elastic bodies.

The observations upon the solar eclipse of 1869, by Morton, Winlock Young, Pickering, Harkness and others, disclosed an important line in the solar spectrum, which corresponds very nearly to 1474 of Kirchhoff's scale, and is now styled 1474 K, or "the corona line." Father Secchi attributed the line to hydrogen ; Dr. Gould thought it identical with the auroral line, and therefore due to some substance which, as stated by Lockyer,§ "may possibly be present in the higher regions of our own atmosphere."

Lockyer ‖ considers the observations as indicating "an enormous envelope of hydrogen, probably in the average twelve minutes high," as well as "the existence of some unknown element extending further from the photosphere even than hydrogen." In the eclipse of 1870, "at the same time that this line was observed to extend to a distance of 20′ from the Sun, the lines of hydrogen were observed eight minutes above the Sun." Here are, therefore, probable evidences of two successive stages of æthereal condensation. It has "been shown by Salet, Schuster and others, that" all the hydrogen lines but F are "due to impurities,"* so that the hydrogen spectrum is harmonically connected with the corona line.

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* Ib., xvii, 109-12, 297-301.

†Ante, ix., 288; Phil. Mag. [4], xxviii, 59.

‡Ante, ix, xiv; xvi, sqq.; P. Mag. [4], vols. 30, 32, 34, 35, 50; [5], 1-6, 10, 11.

§Solar Physics, p. 269.

‖ Ib. p. 413.

It may be well to recapitulate, in this connection, some of the simple equations which serve to connect the energies of solar and terrestrial rotation, planetary revolution, atmospheric limitation, molecular oscillation, cosmical aggregation, and æthereal action :

1. $M : \pi r : : v_\lambda : v_r$ Note 34.
2. $v_r = p\, v_h$ Note 34.
3. $v_\lambda = \left(\frac{gt}{2}\right)_0$ Notes 33, 37.
4. $v_0 = p \left(\frac{gt}{2}\right)_3$ Note 17.

See, also, Proc. Am. Phil. Soc., xii, 392–4 ; xix, 21–5, and Note 16.

45. *Cosmical Significance of the Corona Line.*

Earth being the centre of density in the solar system, its nascent locus should have a time of revolution π times as great as its own, with a semi-axis major of $\pi^{\frac{2}{3}} \times 214.45 = 460.002$ solar semi-diameters. The corresponding wave-length (Note 37,) is 5321.35, as is shown by the following proportion :

$$\log.\ 6441.4 : \log.\ 460.002 : : 7612 : 5321.35.$$

This differs by less than $\frac{1}{152}$ of one per cent. from the geometric wave-length (Note 41), and by less than $\frac{1}{80}$ of one per cent. from Gibbs's measurement of the corona line.

The Systematic Arrangement of the Order Perissodactyla. By E. D. Cope.

(*Read before the American Philosophical Society, April* 15, 1881.)

PERISSODACTYLA.

This, the second great order of the ungulate Mammalia, naturally occupies a position between the *Amblypoda* and the *Artiodactyla*. Its lower forms are more specialized in the structure of the feet than the *Amblypoda*, while its highest types do not reach the perfection of structure seen in the *Artiodactyla*. This is particularly indicated by the form of the astragalus, which has but one, the tibial trochlea, and never displays the distal one characteristic of the cloven-footed families. The *Perissodactyla* occupy, as regards their dentition, a position parallel with the *Artiodactyla*. They are always superior in dental complication to the *Proboscidia* and the suilline *Artiodactyla*, but only one series, that of the horses, reaches the com-

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*Ib. p. 530, foot-note.